OCR
Oxford Cambridge and RSA

Thursday 13 June 2019 – Afternoon

A Level Further Mathematics B (MEI)

Y431/01 Mechanics Minor

Time allowed: 1 hour 15 minutes

You must have:
- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You may use:
- a scientific or graphical calculator

* 7 7 6 9 4 3 8 4 9 4 *

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

1 Dilip and Anna are doing an experiment to find the power at which they each work when running up a staircase at school. The top of the staircase is a vertical distance of 16 m above the bottom of the staircase.

Dilip, who has mass 75 kg, does the experiment first. Anna times him, and finds that he takes 5.6 seconds to run up the staircase.

(a) Find the average power generated by Dilip as he runs up the staircase. **[3]**

Anna, who has mass M kg, then does the same experiment and runs up the staircase in 5.0 seconds. She works out that the average power she has generated is **less than** the corresponding value for Dilip.

(b) Find an inequality satisfied by M. **[2]**

Gareth, who also has mass 75 kg, says that members of his sports club do an exercise similar to this, but they run up a 16 m high sand dune. Gareth can run up the sand dune in 8.4 seconds, but he claims that he generates more power than Dilip.

(c) Give a reason why Gareth's claim could be true. **[1]**

a. Work done = change in energy = mgh

$= 75 \times g \times 16$

$= 11760\text{ J}$

Power $= \dfrac{\text{work done}}{\text{time}} = \dfrac{11760}{5.6} = 2100\text{ W}$

b. $P_A < P_D$

$\dfrac{mgh}{t} < 2100$

$\dfrac{M \times 9.8 \times 16}{5} < 2100$

$\Rightarrow M < 67.0\text{ kg}$

c. The sand will give way as Gareth climbs, so he may do more work in moving the sand than is given by mgh.

2 **(a)** Write down the dimensions of pressure. **[1]**

The SI unit of pressure is the pascal (Pa). 15 Pa is equivalent to Q newtons per square centimetre.

(b) Find the value of Q. **[1]**

Simon thinks the speed, v, of sound in a gas is given by the formula

$v = kP^x d^y V^z$,

where P is the pressure of the gas,

d is the density of the gas,

V is the volume of the gas,

k is a dimensionless constant.

(c) Use dimensional analysis to

- find the values of x and y and
- show that $z = 0$.

[4]

At normal atmospheric pressure the density of air at sea level is $1.29\,\text{kg m}^{-3}$. Under the same conditions the density of helium is $0.166\,\text{kg m}^{-3}$.

(d) Given that the speed of sound in air under these conditions is $340\,\text{m s}^{-1}$, use Simon's formula to find the speed of sound in helium under the same conditions. **[2]**

a. $P = M L^{-1} T^{-2}$

b. $Pa = \frac{N}{m^2}$

as $1m = 1\times10^2 cm$, $1m^2 = 1\times10^4 cm^2$

$\therefore 15\,Pa \equiv Q\ N/1\times10^4 cm^2$

$\Rightarrow Q = 15\times10^{-4} = 0.0015$

c. $[P] = ML^{-1}T^{-2}$

$[d] = ML^{-3}$

$[V] = L^3$

$[v] = LT^{-1}$

Sub into $v = kP^x d^y V^z$:

$$LT^{-1} = (ML^{-1}T^{-2})^x (ML^{-3})^y (L^3)^z$$

Powers on RHS must = powers on LHS:

T: $-1 = -2x$

$x = \frac{1}{2}$

M: $0 = x + y$

$y = -x$

$y = -1/2$

L: $1 = -x - 3y + 3z$

$1 = -1/2 + 3/2 + 3z$

$\Rightarrow z = 0$ as required

d. $k = \dfrac{v}{d^{-1/2}}$ (as pressure is the same for both gasses, it will cancel)

$k = 340 \div \sqrt{\dfrac{1}{1.29}}$
$= 386.17$

$\therefore$ for helium : $v = 386.17 \times \sqrt{\dfrac{1}{0.166}}$
$= 948 \text{ ms}^{-1}$

3 Two identical uniform rectangular laminas, P and Q, each having length ka and width a are fixed together, in the same plane, to form a lamina R.
With reference to coordinate axes, the corners of P are at (0, 0), (ka, 0), (ka, a) and (0, a) and the corners of Q are at (ka, 0), ($ka + a$, 0), ($ka + a$, ka) and (ka, ka), as shown in Fig. 3.

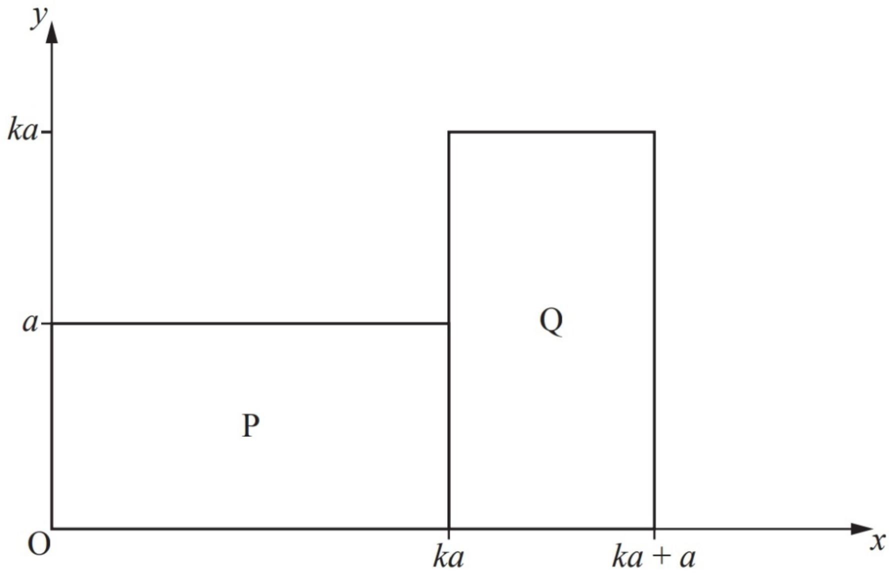


Fig. 3

Determine the range of values of k for which the centre of mass of R lies outside the boundary of R. **[7]**

$\bar{x}$:

COM of P $= \frac{ka}{2}$

COM of Q $= ka + \frac{a}{2}$

$\therefore \Sigma$ COM $= \frac{1}{2}\left(\frac{ka}{2} + ka + \frac{a}{2}\right)$

$= \frac{3}{4}ka + \frac{1}{4}a$

$\bar{x} < ka$ (in order to be outside R)

$\frac{3}{4}k\cancel{a} + \frac{1}{4}\cancel{a} < k\cancel{a}$

$\frac{1}{4} < \frac{1}{4}k \quad \Rightarrow k > 1$

$\bar{y}$:

COM of P : $\frac{a}{2}$

COM of Q : $\frac{ka}{2}$

$\therefore \Sigma COM = \frac{1}{2}\left(\frac{a}{2} + \frac{ka}{2}\right)$

$= \frac{1}{4}ka + \frac{1}{4}a$

$\bar{y} > a$ (in order to be outside R)

$\frac{1}{4}k\cancel{a} + \frac{1}{4}\cancel{a} > \cancel{a}$

$\frac{1}{4}k > \frac{3}{4}$

$k > 3$

So $k > 3$ only

4 Two model railway trucks, A of mass 0.1 kg and B of mass 0.2 kg, are constrained to move on a smooth straight level track.
Initially B is stationary and A is moving towards B with speed $u\,\mathrm{m\,s^{-1}}$ before they collide. The coefficient of restitution between A and B is e.

(a) Find the speed of A and the speed of B after the collision, giving your answers in terms of e and u. **[5]**

(b) Show that the loss of kinetic energy in the collision is $\frac{1}{30}u^2(1-e^2)$. **[2]**

(c) For the case in which the **loss** of kinetic energy is **least**

- state the value of e
- state the loss in kinetic energy
- describe the subsequent motion of the trucks. **[3]**

(d) For the case in which the **loss** of kinetic energy is **greatest**

- state the value of e
- state the loss in kinetic energy
- describe the subsequent motion of the trucks. **[3]**

a. Conservation of momentum:

$$0.1u + 0.2\times 0 = 0.1a + 0.2b \quad (1)$$

Newton's restitution:

$$e = \frac{b-a}{u-0} \Rightarrow b - a = eu$$
$$b = eu + a \quad (2)$$
$$a = b - eu \quad (3)$$

Sub (2) into (1):

$$0.1u = 0.1a + 0.2(eu + a)$$
$$0.1u - 0.2eu = 0.3a$$
$$a = \frac{1}{3}u(1-2e) \quad (4)$$

(×10 and rearrange for a)

Sub ④ into ③:

$$\frac{1}{3}u - \frac{2}{3}eu = b - eu$$

$$b = \frac{1}{3}u(1+e)$$

b. loss = initial − final

$$= \frac{1}{2}\times 0.1\times u^2 - \left(\frac{1}{2}\times 0.1\left(\frac{1}{3}u(1-2e)\right)^2 + \frac{1}{2}\times 0.2\left(\frac{1}{3}u(1+e)\right)^2\right)$$

$$= \frac{1}{20}u^2 - \left(\frac{1}{180}u^2(1+4e^2-4e) + \frac{1}{90}u^2(1+e^2+2e)\right)$$

$$= \frac{1}{20}u^2 - \left(\frac{1}{60}u^2 + \frac{1}{30}u^2e^2\right)$$

$$= \frac{1}{30}u^2 - \frac{1}{30}u^2e^2$$

$$= \frac{1}{30}u^2(1-e^2) \text{ as required}$$

c. Loss is least at $e = 1$

$$\text{loss} = \frac{1}{30}u^2(1-1) = 0\ \text{J}$$

A and B will move away from each other.

$a = \frac{1}{3}u\ \text{ms}^{-1}$ and $b = \frac{2}{3}u\ \text{ms}^{-1}$

d. Loss is greatest at $e = 0$

$$\text{Loss} = \frac{1}{30}u^2(1-0) = \frac{1}{30}u^2 \text{ J}$$

A and B coalesce and move with speed $\frac{1}{3}u$ ms^{-1} in the direction of u.

5 Jack and Jemima are pulling a boat along a straight level canal.
The resistance to the motion of the boat is modelled as constant and equal to 1200 N.
Jack and Jemima walk in the same direction on paths on opposite sides of the canal. They each walk forwards at the same steady speed, keeping level with each other so that the distance between them is always 6 m. Jack and Jemima each pull a long light inextensible rope attached to the boat; initially they hold their ropes so the distance from each of them to the boat is 5 m, as shown in Fig. 5.1.

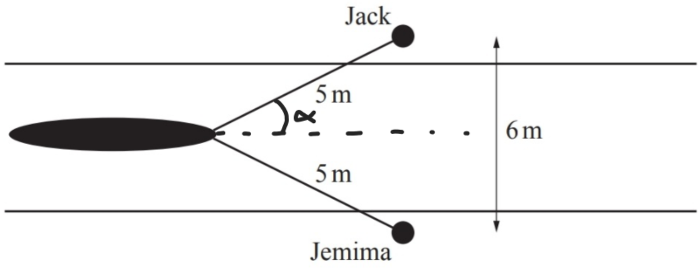


Fig. 5.1

(a) Explain why the tension will be the same in each rope. **[1]**

(b) Find the tension in each rope. **[3]**

Jemima then gradually releases more rope, so that the distance between her and the boat is 7 m. Jack and Jemima continue to walk at the same steady speed along the paths, but the position of the boat changes so that Jemima's rope makes an angle of θ with the path and Jack's rope makes an angle of ϕ with the path, as shown in Fig. 5.2.

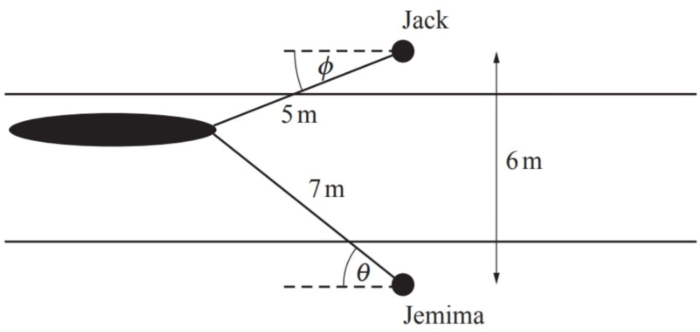


Fig. 5.2

(c) • Show that $\sin\phi = \frac{1}{5}$.
• Show that $\sin\theta = \frac{5}{7}$. **[2]**

(d) Find the tension in each rope in this new equilibrium position. **[5]**

(e) Without further calculation, state the effect on the tensions in the ropes if Jack now lengthens his rope to 7 m, the same length as Jemima's rope. **[2]**

(f) Suggest how the modelling assumption used in this question could be improved. **[1]**

a. The system is symmetrical.

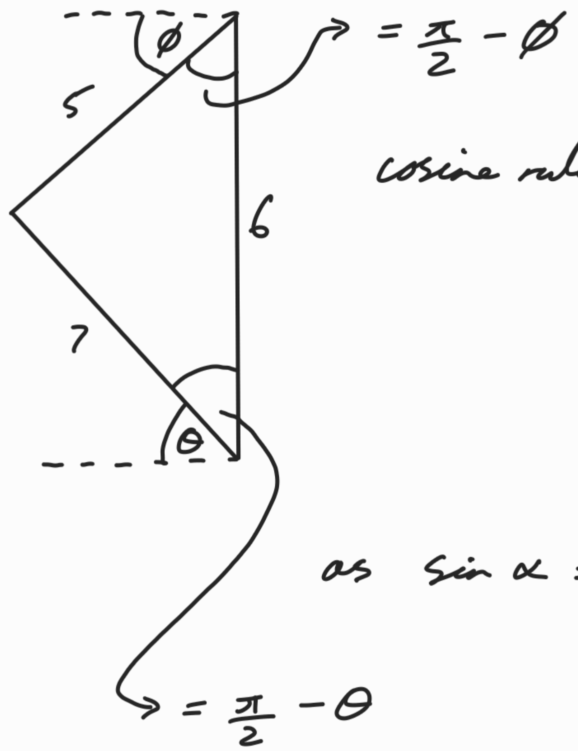
b. $\sin\alpha = \frac{3}{5} = 0.6 \quad \Rightarrow \cos\alpha = \frac{4}{5}$

Constant speed $\Rightarrow$ Driving force = resistance

$\Rightarrow 2T\cos\alpha = 1200$

$T = 750\text{ N}$

c.



cosine rule : $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$$\cos\left(\frac{\pi}{2} - \phi\right) = \frac{5^2 + 6^2 - 7^2}{2 \times 5 \times 6}$$

$$= \frac{1}{5}$$

as $\sin \alpha = \cos\left(\frac{\pi}{2} - \alpha\right)$, $\sin \phi = \frac{1}{5}$ as req.

cosine rule : $\cos\left(\frac{\pi}{2} - \theta\right) = \frac{6^2 + 7^2 - 5^2}{2 \times 6 \times 7}$

$$= \frac{5}{7}$$

$\Rightarrow \sin \theta = \frac{5}{7}$ as required

d. Resolve in direction of motion :

(T_1 in Jack's rope
T_2 in Jemima's)

$$\cos \phi = \frac{\sqrt{5^2 - 1^2}}{5} = \frac{\sqrt{24}}{5}$$

$$\cos \theta = \frac{\sqrt{7^2 - 5^2}}{7} = \frac{\sqrt{24}}{7}$$

$$T_1 \cos \phi + T_2 \cos \theta = 1200$$

$$T_1 \frac{\sqrt{24}}{5} + T_2 \frac{\sqrt{24}}{7} = 1200 \quad (1)$$

Resolve perpendicular to direction of motion:

$$T_1 \sin\phi = T_2 \sin\theta$$

$$\frac{1}{5}T_1 = \frac{5}{7}T_2$$

$$T_1 = \frac{25}{7}T_2$$

Sub into ①:

$$T_2 \times \frac{25}{7} \times \frac{\sqrt{24}}{5} + T_2\frac{\sqrt{24}}{7} = 1200$$

$$T_2 \times \frac{6\sqrt{24}}{7} = 1200$$

$\underline{T_2 = 286\text{ N}}$ (3sf)

$$\Rightarrow T_1 = \frac{25}{7} \times 285.77$$

$\underline{T_1 = 1020\text{ N}}$ (3sf)

e. When the new equilibrium is reached, the system will be symmetrical again, so the tensions will be equal. They will be less than 750 N, as the angle will be less.

f. Resistance to motion may be different when the boat is closer to the bank in (d).

6 A uniform solid cylinder, L, has base radius 5 cm, height 24 cm and mass 5 kg.
L is placed on a rough plane inclined at an angle α to the horizontal, as shown in Fig. 6.

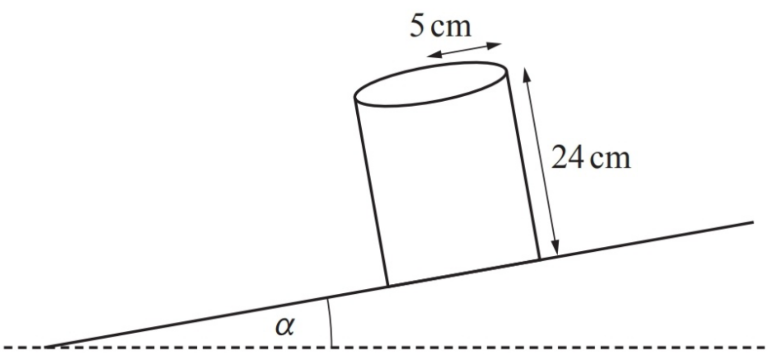


Fig. 6

(a) On the copy of Fig. 6 in the Printed Answer Booklet mark the forces acting on L. **[1]**

The coefficient of friction between L and the plane is 0.3. Initially α is 15°.

(b) Show that L rests in equilibrium on the plane. **[4]**

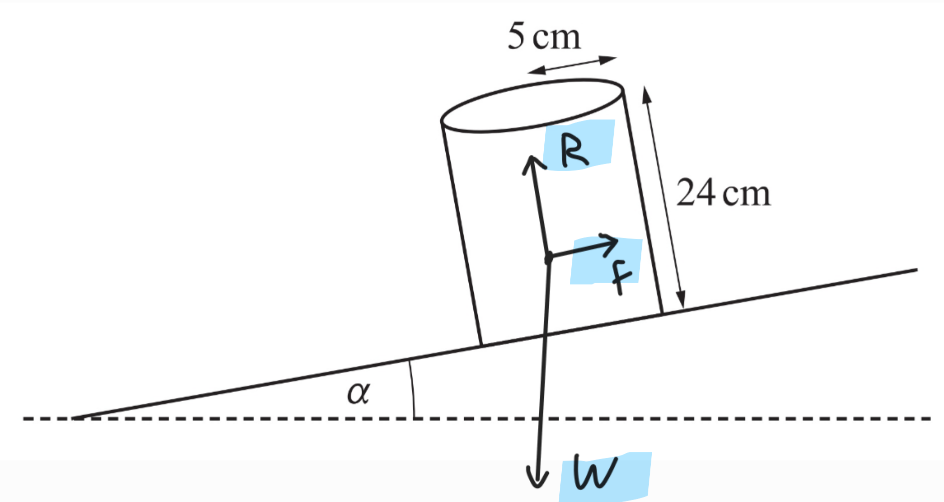
A couple is applied to L. It is given that L will topple if the couple is applied in an anticlockwise sense, but L will not topple if the couple is applied in a clockwise sense.

(c) Find the range of possible values of the magnitude of the couple. **[4]**

The couple is now removed and the plane is slowly tilted so that α increases.

(d) Determine whether L topples first without sliding or slides first without toppling. **[3]**

a.



b. Parallel to the plane :

$W \sin \alpha = 5 \times 9.8 \times \sin 15 = 12.7$

$f_{max} = \mu R = 0.3 \times W \cos 15$
$= 0.3 \times 5 \times 9.8 \times \cos 15$
$= 14.2$

$12.7 < 14.2$
downward force < maximum friction
$\Rightarrow$ does not slip $\Rightarrow$ equilibrium

Toppling angle $= \tan^{-1}\left(\frac{5}{12}\right) = 22.6°$

$22.6 > 15$
$\Rightarrow$ does not topple $\Rightarrow$ equilibrium

c. Take moments about centre of base of cylinder :
let magnitude of couple = c

If C is applied anticlockwise :

force × perpendicular distance

$C + 5g \times 12 \sin 15 > 5g \times 5 \cos 15$
$c > 236.7 - 152.2$
$c > 84.5$

Continued on next page

If C is applied clockwise:

$C \leq 5g \times 12 \sin 15 + 5g \times 5 \cos 15$
$C \leq 236.7 + 152.2$
$C \leq 389$

$\Rightarrow 84.5 < C \leq 389$

d. Slides at $\tan \alpha = \mu = 0.3$
$\alpha = 16.7^\circ$

Topples at $\tan \alpha = 5/12$
$\alpha = 22.6^\circ$

$\therefore$ Slides first.